Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

[Total No. of Questions: 09] [Total No. of Pages: 2]

Uni. Roll No. ....

MORNING

10 MAY 2023

Program/ Course:B.Tech.(Sem. Third)
Name of Subject: Engineering Mathematics III
Subject Code:BSEC-101
Paper ID: 16030
Scientific calculator is Not Allowed

Time Allowed: 03 Hours **Max. Marks: 60**

NOTE:

1) Parts A and B are compulsory
2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
3) Any missing data may be assumed appropriately

Section – A **[Marks:02 each]**

Q1.

(a) Separate the function $cos(x+\ iy)$ into real and imaginary parts.

(b) State and prove Linear property of Laplace Transform.

(c) Evaluate $\oint_c \frac{1}{z+4} dz$ where c is the circle $|z| = 2$.

(d) Solve $(y-z)\frac{\partial z}{\partial x} + (x-y)\frac{\partial z}{\partial y} = z - x$.

(e) Write Legendre's differential equation **.**

(f) Classify the following partial differential equation:

$$2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 3\frac{\partial^2 u}{\partial x^2} = 0$$

Section – B **[Marks:04 each]**

Q2. Show that the function $u = \frac{1}{2}log(x^2 + y^2)$ is harmonic.

Q3 Evaluate the Laplace transform of the function $f(t) = t^2 e^t sin4t$.

Q4. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$.

Q5 If $\cos h(u + iv) = x + iy$, prove that $\frac{x^2}{cosh^2 u} + \frac{y^2}{sinh^2 u} = 1.$

Q6. Prove that $x\,J'_n(x) = n\,J_n(x) - x\,J_{n+1}(x)$ where $J_n(x)$ is Bessel function of first kind of order n.

Q7. If $2\,cos\theta = x + \frac{1}{x}$, then prove that $x^r + \frac{1}{x^r} = 2\cos r\theta$.

Section – C **[Marks:12 each]**

Q8. Solve the following differential equation using Laplace transform:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = t,\ y(0) = 1,\ y'(0) = 0.$$

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$, using Residue theorem

Q9) A tightly stretched flexible string has its ends fixed at x = 0 and x = l. At time t = 0 the string is given a shape defined by $y = \mu x(l - x)$, μ is a constant and then released . Find the displacement y(x,t) of any point x of the string at any time t > 0

OR

Solve in series the differential equation :

$$\frac{d^2y}{dx^2} + xy = 0.$$

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